

Systematic $U(1)_{B-L}$ Extensions of Loop-Induced Neutrino Mass Models with Dark Matter

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Abstract

We study the gauged $U(1)_{B-L}$ extensions of the models for neutrino masses and dark matter. In this class of models, tiny neutrino masses are radiatively induced through the loop diagrams, while the origin of the dark matter stability is guaranteed by the remnant of the gauge symmetry. Depending on how the lepton number is violated in the neutrino mass diagrams, these models are systematically classified. We present a complete list for the one-loop \mathbb{Z}_2 and the two-loop \mathbb{Z}_3 neutrino mass models as examples of the classification. These underlying gauge symmetries and its breaking patterns can be probed at future high energy colliders by looking at the width of the new gauge boson.

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I. INTRODUCTION

The standard model (SM) of particle physics has become established after the discovery of the 125 GeV Higgs boson [1, 2]. Nonetheless, there are still some remaining puzzles which the SM is unable to address. In particular, generating the small neutrino masses measured by the neutrino-oscillation experiments [3–6] and accommodating cosmic dark matter (DM) inferred from the astronomical observations [7–11] are phenomenologically important issues. The simplest solution to explain the tiny neutrino masses is the canonical seesaw mechanism with super heavy right-handed neutrinos [12–14], while its verification by experiments may be difficult. On the other hand, various DM candidates such as axion, weakly interacting massive particles, asymmetric DM, strongly interacting massive particles and wimpzilla are suggested, and the DM mass scale spreads over very wide range from 10^{-15} GeV to 10^{15} GeV [15].

Models with radiative neutrino masses are among the most economical scenarios that can resolve the above two issues. This is the case with models in which neutrino mass is induced by quantum effects and a DM candidate is also incorporated as a necessary component. Since the DM particle is responsible to run in the loop diagram in order to get the neutrino masses, phenomenology of neutrinos and DM is strongly correlated. Representative models possessing these ingredients include Ma’s scotogenic models at one-loop [16], two-loop level [17], the Krauss-Nasri-Trodden (KNT) model at three-loop level [18] and another three-loop models proposed by Aoki, Kanemura, and Seto [19] and, by Gustafsson, No and Rivera [20]. In each of these models, an *ad hoc* discrete symmetry \mathbb{Z}_2 or \mathbb{Z}_3 is imposed to forbid tree-level neutrino masses and stabilize the DM candidate, but the origin of the symmetry is left unknown. One possibility of the origin of this symmetry is accidental symmetry. If one extends the SM with higher $SU(2)_L$ multiplets such as quintuplets or septuplets, an accidental \mathbb{Z}_2 symmetry appears and stabilizes the DM candidate [21–23].¹ Extensions of the models with radiative neutrino mass generation along this line and phenomenology of them have been explored in Ref. [25–27]. Another attractive possibility is that this discrete symmetry originates from a continuous symmetry that is spontaneously broken at some high energy scale by a scalar field with non-zero vacuum expectation value (VEV). If the discrete symmetry is originated from the gauge symmetry, this is the so-called Krauss-Wilczek mechanism [28], and has been applied to some neutrino mass models [29]. The spontaneous breaking of a global symmetry can also leaves a \mathbb{Z}_2 symmetry, which is relevant to the domain wall production in axion models. This residual symmetry has also been used to construct a radiative seesaw model [30].

In this paper, we study the gauged $U(1)_{B-L}$ extension of the models described above. The \mathbb{Z}_N discrete symmetry can be realized as a remnant of the $U(1)_{B-L}$ gauge symmetry. Although this kind of extension has been already discussed in the literature [31–33], our aim is to cover all possible models systematically and focus on classification of the extended models. We do not give detailed numerical analysis of each model at this stage. As we will see, these $U(1)_{B-L}$ extended models generally encounter gauge anomalies. Hence, we also discuss a way of anomaly cancellations by adding some extra fermions. Moreover, we will show that measurements of the width of the new gauge boson can be a nice discriminant of these models.

¹ A large isospin *scalar* multiplet leads a lower cutoff scale, which might disturb the DM stability [24].

The structure of the paper is as follows. In the next section, we present a systematic approach to classify the gauged $U(1)_{B-L}$ models for the neutrino mass generation and the DM stability. In Sec. III, we deal with the gauge anomaly cancellations for some $U(1)_{B-L}$ extended models, and the required number of the extra fermions and their B–L charges are given in Appendix as examples. The discrimination of these models via collider signals is discussed in Sec. IV. We conclude and summarize our study in Sec. V.

II. $U(1)_{B-L}$ EXTENSIONS

We demonstrate the $U(1)_{B-L}$ extension of the radiative neutrino mass models at one-loop, two-loop and three-loop level [16–18]. The discrete \mathbb{Z}_N symmetry can be derived as a residual symmetry of the $U(1)_{B-L}$ gauge symmetry as we will see below. In the following investigation, the models we will discuss include gauge anomalies in general. However, we do not pay attention to anomaly cancellations for the moment since these anomalies can be cancelled by introducing some pairs of vector-like fermions (under the SM gauge group) as have been examined in Ref. [34–36]. This cancellation will be addressed in the next section.

A. One-loop \mathbb{Z}_2 Model

First, we consider the $U(1)_{B-L}$ extension of the one-loop model with the \mathbb{Z}_2 symmetry [16].² In the original one-loop model, three right-handed singlet fermions N_R and one inert doublet scalar $\eta = (\eta^+, \eta^0)^T$ are added to the SM. In addition, the \mathbb{Z}_2 parity is assigned to be odd for the new particles and even for the SM particles. The neutral lightest \mathbb{Z}_2 odd particle which is either the lightest right-handed fermion or the neutral component of the inert scalar can be a DM candidate. Also, at least two generations of the right-handed fermion are needed to explain the observed neutrino masses and mixings. The required interactions to generate neutrino masses in the original one-loop model are written down as

$$\mathcal{L} \supset \bar{E} N_R \tilde{\eta}, \quad \bar{N}_R^c N_R, \quad (\Phi^\dagger \eta)^2, \quad + \text{h.c.}, \quad (1)$$

where E (Φ) is the SM lepton (Higgs) doublet, $\tilde{\eta} = i\tau_2 \eta^*$ with τ_2 being the usual second Pauli matrix, and $N_R^c \equiv (N_R)^c$ denotes the charge conjugate of N_R .

In order to achieve the $U(1)_{B-L}$ extension, we adopt the particle contents and the charge assignments as shown in Table I. Two new complex singlet scalars χ and σ are added to the original one-loop model. Although the B–L charges of quarks are not displayed in Table I, these are fixed appropriately as usual. Hereafter we assume that only σ among the new scalar fields develops a VEV and triggers the $U(1)_{B-L}$ symmetry breaking. The function of χ is to induce the third term in Eq. (1) effectively through the mixing between η^0 and χ when the $U(1)_{B-L}$ symmetry is broken. This is because $(\Phi^\dagger \eta)^2$ cannot be allowed in the $U(1)_{B-L}$ extended model

² Phenomenology of this model has been studied in Ref. [37–42] for example.

	E	Φ	N_R^c	η	χ	σ
$SU(2)_L$	2	2	1	2	1	1
$U(1)_Y$	$-1/2$	$1/2$	0	$1/2$	0	0
$U(1)_{B-L}$	-1	0	Q_N	Q_η	Q_χ	Q_σ
Spin J	$1/2$	0	$1/2$	0	0	0

TABLE I: Charge assignments of the fermions and scalars in the $U(1)_{B-L}$ extended one-loop model, where Q_N , Q_η , Q_χ and Q_σ ($Q_\chi, Q_\sigma \neq 0$) can be determined properly as discussed in the context.

since Φ must be neutral while η should be charged under the $U(1)_{B-L}$ symmetry. In this set up, the necessary interactions to induce neutrino masses in the extended one-loop model are listed by

$$\mathcal{L} \supset \bar{E} N_R \tilde{\eta} \begin{vmatrix} \overline{N_R^c} N_R & (\Phi^\dagger \eta) \chi & (\Phi^\dagger \eta) \chi^* \\ \overline{N_R^c} N_R \sigma & (\Phi^\dagger \eta) \chi \sigma & (\Phi^\dagger \eta) \chi^* \sigma \\ \overline{N_R^c} N_R \sigma^* & (\Phi^\dagger \eta) \chi \sigma^* & (\Phi^\dagger \eta) \chi^* \sigma^* \end{vmatrix} \begin{vmatrix} \chi^2 \sigma \\ \chi^2 \sigma^* + \text{h.c.} \end{vmatrix} \quad (2)$$

At least one element from each column has to be selected in the extended model. Since these terms must be invariant under the $U(1)_{B-L}$ transformation, the simultaneous equations of the unknown charges ($Q_N, Q_\eta, Q_\chi, Q_\sigma$) are obtained, and can be solved analytically. Then once the B–L charges are found, one should check whether the other unselected terms are permitted by the assignment. Notice that the scalar interactions in the last column of Eq. (2) automatically exclude the χ linear terms $\sigma^n \chi$ and $(\sigma^*)^n \chi$ ($n = 1, 2, 3$) which should not be allowed to obtain a remnant \mathbb{Z}_2 symmetry after the $U(1)_{B-L}$ symmetry breaking.

The topological diagrams of the neutrino mass generation are shown in Fig. 1, where the square symbol indicates the possible B–L breaking vertex. Depending on the B–L charge assignment, which vertex violates the lepton number would change. In other words, the insertion of $\langle \sigma \rangle$ is decided by the lepton numbers of the new particles in the diagram. The possible B–L charge assignments can be summarized in Table II, where the assignments obtained by the replacement $\chi^* \rightarrow \chi$ or $\sigma^* \rightarrow \sigma$ are regarded as the same model. The assignments A1 and A4 involve two kinds of terms which induce the mixing between η^0 and χ , namely $(\Phi^\dagger \eta) \chi^*$ and $(\Phi^\dagger \eta) \chi \sigma$. The

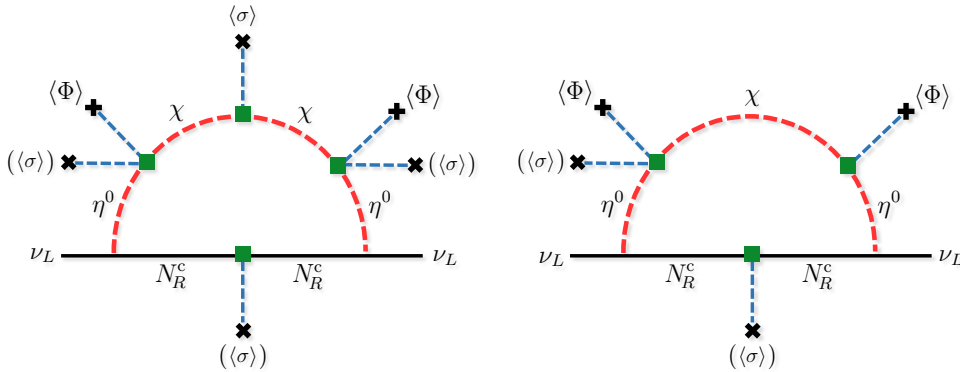


FIG. 1: Topology of the neutrino mass generation in the extended one-loop model where the square symbol implies the possible B–L breaking vertex.

	Q_N	Q_η	Q_χ	Q_σ	necessary interactions
A1	1/2	1/2	1/2	-1	$\bar{E}N_R\tilde{\eta}, \bar{N}_R^c N_R\sigma^*, (\Phi^\dagger\eta)\chi^*, (\Phi^\dagger\eta)\chi\sigma, \chi^2\sigma$
A2	1/4	3/4	1/4	-1/2	$\bar{E}N_R\tilde{\eta}, \bar{N}_R^c N_R\sigma^*, (\Phi^\dagger\eta)\chi^*\sigma, \chi^2\sigma$
A3	-1/2	3/2	-1/2	1	$\bar{E}N_R\tilde{\eta}, \bar{N}_R^c N_R\sigma^*, (\Phi^\dagger\eta)\chi\sigma^*, \chi^2\sigma$
A4	0	1	1	-2	$\bar{E}N_R\tilde{\eta}, \bar{N}_R^c N_R, (\Phi^\dagger\eta)\chi^*, (\Phi^\dagger\eta)\chi\sigma, \chi^2\sigma$
A5	0	1	1/3	-2/3	$\bar{E}N_R\tilde{\eta}, \bar{N}_R^c N_R, (\Phi^\dagger\eta)\chi^*\sigma, \chi^2\sigma$

TABLE II: Possible B–L charge assignment for new particles in the $U(1)_{B-L}$ extended one-loop model. In each of these assignments, the last column contains the necessary interactions to generate neutrino masses at one-loop level.

other assignments include only one mixing term. The way of lepton number violation is different for each assignment. For instance, in the case of A1, A2 and A3, since all the new particles carry non-zero B–L charge, both of the fermion and scalar lines in Fig. 1 violate lepton number. On the other hand, for the assignments A4 and A5, since the new fermion N_R^c has no B–L charge, lepton number violation occurs in the scalar line only.

For the assignments A1-A3, the remnant symmetry $\mathbb{Z}_2 \equiv (-1)^{NQ_{B-L}}$ is identical to the one in the original Ma’s one-loop model [16] when the $U(1)_{B-L}$ symmetry is broken, where $NQ_{B-L} (\equiv 2Q_{B-L}/Q_\sigma)$ focusing on lepton sector. While for the assignments A4 and A5, $\mathbb{Z}_2 \equiv (-1)^{NQ_{B-L}+2J}$ is the same as the original model, where J is the spin of the particle. This is because $(-1)^{2J}$ is an accidental \mathbb{Z}_2 symmetry, and then a product of \mathbb{Z}_2 symmetries leads to another \mathbb{Z}_2 symmetry.

B. Two-loop \mathbb{Z}_3 Model I

The two types of the two-loop neutrino mass models with \mathbb{Z}_3 symmetry have been proposed in Ref. [17], and phenomenology of these models have been discussed in the literature [43, 44]. In the first two-loop model, a pair of $SU(2)_L$ doublet vector-like fermion $\Sigma = (\Sigma^+, \Sigma^0)^T$ and singlet vector-like fermion ψ , and three complex singlet scalars χ are added to the SM.³ The \mathbb{Z}_3 charge is assigned as $\omega = e^{2\pi i/3}$ or ω^* for the new particles and trivial for the SM particles. The lightest scalar χ can serve as DM candidate, and a fermionic DM candidate given by the lightest mass eigenstate composed of Σ^0 and ψ may not be suitable for a DM candidate since the elastic cross section via Z boson with nuclei is strongly constrained by direct detection searches [45]. The required interactions for generating neutrino masses in the original two-loop model I are listed by

$$\mathcal{L} \supset \bar{E}\Sigma_R\chi^*, \quad \bar{\Sigma}_R\Sigma_L, \quad \bar{\psi}_R\psi_L, \quad \bar{\Sigma}_L\psi_R\tilde{\Phi}, \quad \bar{\psi}_L^c\psi_L\chi, \quad \chi^3, \quad + \text{h.c.} \quad . \quad (3)$$

To accomplish the $U(1)_{B-L}$ extension, here we simply introduce one singlet complex scalar σ

³ Conversely, one can add three pairs of $SU(2)_L$ doublet and singlet vector-like fermion, and one singlet complex scalar to induce neutrino masses consistent with the observations.

	E	Φ	Σ_L	Σ_R^c	ψ_L	ψ_R^c	χ	σ
$SU(2)_L$	2	2	2	2	1	1	1	1
$U(1)_Y$	$-1/2$	$1/2$	$-1/2$	$1/2$	0	0	0	0
$U(1)_{B-L}$	-1	0	Q_{Σ_L}	Q_{Σ_R}	Q_{ψ_L}	Q_{ψ_R}	Q_χ	Q_σ
Spin J	$1/2$	0	$1/2$	$1/2$	$1/2$	$1/2$	0	0

TABLE III: Charge assignments of the fermions and scalars in the $U(1)_{B-L}$ extended two-loop model I where the unknown B–L charges can be determined by the same strategy in the previous subsection.

to the original two-loop model I as displayed in Table III.⁴ The necessary interactions to produce neutrino masses in the extended two-loop model I are given by

$$\mathcal{L} \supset \bar{E} \Sigma_R \chi^* \left| \begin{array}{c} \bar{\Sigma}_R \Sigma_L \\ \bar{\Sigma}_R \Sigma_L \sigma \\ \bar{\Sigma}_R \Sigma_L \sigma^* \end{array} \right| \left| \begin{array}{c} \bar{\psi}_R \psi_L \\ \bar{\psi}_R \psi_L \sigma \\ \bar{\psi}_R \psi_L \sigma^* \end{array} \right| \left| \begin{array}{c} \bar{\Sigma}_L \psi_R \tilde{\Phi} \\ \bar{\Sigma}_R \psi_L \tilde{\Phi} \end{array} \right| \left| \begin{array}{cc} \bar{\psi}_L^c \psi_L \chi & \bar{\psi}_R^c \psi_R \chi \\ \bar{\psi}_L^c \psi_L \chi^* & \bar{\psi}_R^c \psi_R \chi^* \end{array} \right| \left| \begin{array}{c} \chi^3 \sigma \\ \chi^3 \sigma^* \end{array} \right| + \text{h.c.} . \quad (4)$$

At least one unit in each column should be chosen to construct the extended model with neutrino masses. The topological diagrams of the neutrino mass generation are shown in Fig. 2. By the same strategy illustrated in the previous subsection, the possible charge assignments can be summarized in Table IV. There are nine different ways (B1-B9) of assignment. In the assignments B5-B7, the Yukawa interaction $\bar{\Sigma}_R \psi_L \tilde{\Phi}$ is also allowed to generate the neutrino mass diagram (e.g. see right-panel of Fig. 2). After the $U(1)_{B-L}$ symmetry breaking, the \mathbb{Z}_3 charge defined by $(-1)^{NQ_{B-L}+2J}$ remains, and it is consistent with the \mathbb{Z}_3 charge in the original two-loop model I.

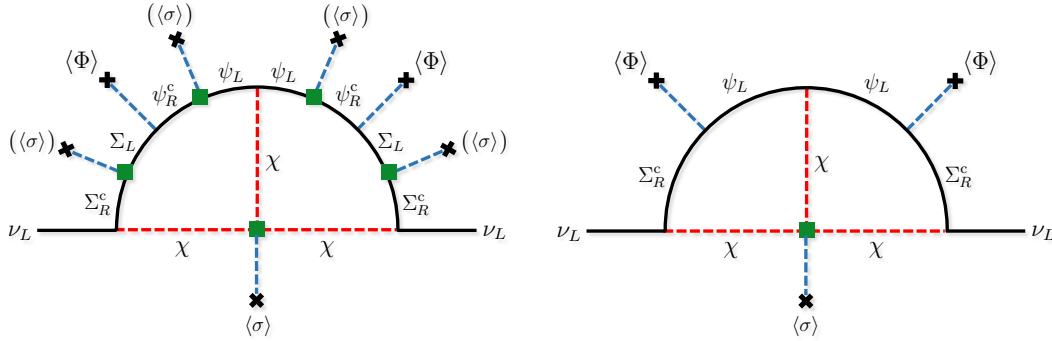


FIG. 2: Topology of the neutrino mass generation in the extended two-loop model I.

C. Two-loop \mathbb{Z}_3 Model II

For the second original two-loop model built in Ref. [17], one $SU(2)_L$ doublet scalar η is added instead of $SU(2)_L$ doublet vector-like fermion Σ in the first two-loop model. The other parts are

⁴ In order to reproduce the original two-loop model I exactly, we also assume that three complex scalar particles χ share the same B–L charge.

	Q_{Σ_L}	Q_{Σ_R}	Q_{ψ_L}	Q_{ψ_R}	Q_χ	Q_σ	necessary interactions
B1	-7/15	13/15	-1/15	7/15	2/15	-2/5	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma, \overline{\psi}_R\psi_L\sigma, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B2	-7/9	7/9	-1/9	7/9	2/9	-2/3	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L, \overline{\psi}_R\psi_L\sigma, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B3	-1/9	7/9	-1/9	1/9	2/9	-2/3	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma, \overline{\psi}_R\psi_L, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B4	-5/9	11/9	1/9	5/9	-2/9	-2/3	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma, \overline{\psi}_R\psi_L\sigma, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$
B5	-7/3	1/3	-1/3	7/3	2/3	-2	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma^*, \overline{\psi}_R\psi_L\sigma, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B6	5/3	1/3	-1/3	-5/3	2/3	-2	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma, \overline{\psi}_R\psi_L\sigma^*, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B7	-1/3	1/3	-1/3	1/3	2/3	-2	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L, \overline{\psi}_R\psi_L, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma$
B8	1/3	5/3	1/3	-1/3	-2/3	-2	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L\sigma, \overline{\psi}_R\psi_L, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$
B9	-5/3	5/3	1/3	5/3	-2/3	-2	$\overline{E}\Sigma_R\chi^*, \overline{\Sigma}_R\Sigma_L, \overline{\psi}_R\psi_L\sigma, \overline{\Sigma}_L\psi_R\tilde{\Phi}, \overline{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$

TABLE IV: Possible B–L charge assignment for new particles in the $U(1)_{B-L}$ extended two-loop model I. The last column contains the necessary interactions to generate neutrino masses at two-loop level. Notice that the Yukawa interaction $\overline{\Sigma}_R\psi_L\tilde{\Phi}$ is also allowed to produce the neutrino mass diagram in the assignments B5-B7.

nothing different from the two-loop model I. The required interactions for generating neutrino masses in the original two-loop model II are listed as

$$\mathcal{L} \supset \overline{E}\psi_R\tilde{\eta}, \quad \overline{\psi}_R\psi_L, \quad \Phi^\dagger\eta\chi^*/\Phi^\dagger\eta\chi^2, \quad \overline{\psi}_L^c\psi_L\chi, \quad \chi^3, \quad + \text{h.c.}, \quad (5)$$

Similarly, we consider only one singlet complex scalar σ to reproduce the original two-loop model II. The quantum numbers of the particles in the $U(1)_{B-L}$ extended two-loop model II are displayed in Table V. The necessary interactions to induce neutrino masses are then given by

$$\mathcal{L} \supset \overline{E}\psi_R\tilde{\eta} \begin{vmatrix} \overline{\psi}_R\psi_L \\ \overline{\psi}_R\psi_L\sigma \\ \overline{\psi}_R\psi_L\sigma^* \end{vmatrix} \begin{vmatrix} \Phi^\dagger\eta\chi^* \\ \Phi^\dagger\eta\chi^*\sigma \\ \Phi^\dagger\eta\chi^*\sigma^* \end{vmatrix} \begin{vmatrix} \overline{\psi}_L^c\psi_L\chi & \overline{\psi}_R^c\psi_R\chi \\ \overline{\psi}_L^c\psi_L\chi^* & \overline{\psi}_R^c\psi_R\chi^* \end{vmatrix} \begin{vmatrix} \chi^3\sigma \\ \chi^3\sigma^* \end{vmatrix} + \text{h.c.}. \quad (6)$$

At least one component in each column is needed to produce neutrino masses. There are two types of the topological diagrams for neutrino mass generation as presented in Fig. 3. By using the same procedure, we find nine possible charge assignments as shown in Table VI. However, only three assignments C5, C6 and C9 reproduce the original two-loop model II since the term $\Phi^\dagger\eta\chi^2$ which has been included in the original model is not allowed for the other cases. Although

	E	Φ	ψ_L	ψ_R^c	η	χ	σ
$SU(2)_L$	2	2	1	1	2	1	1
$U(1)_Y$	-1/2	1/2	0	0	1/2	0	0
$U(1)_{B-L}$	-1	0	Q_{ψ_L}	Q_{ψ_R}	Q_η	Q_χ	Q_σ
Spin J	1/2	0	1/2	1/2	0	0	0

TABLE V: Charge assignments of the fermions and scalars in the $U(1)_{B-L}$ extended two-loop model II where the unknown B–L charges can be fixed by using the same procedure in subsection A.

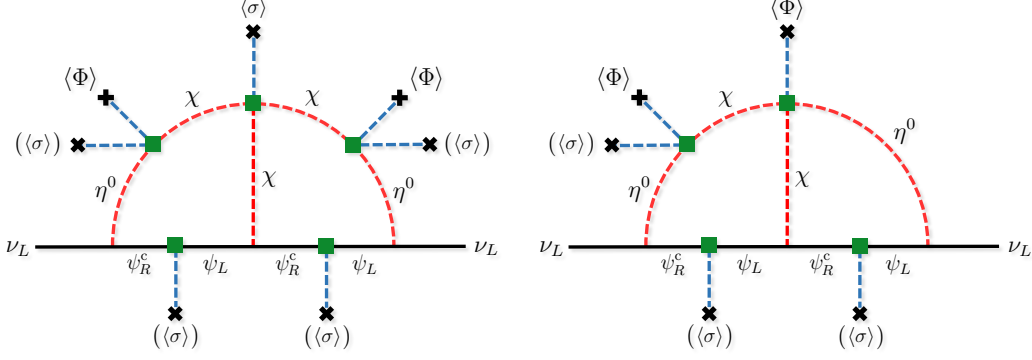


FIG. 3: Topology of the neutrino mass generation in the extended two-loop model II.

	Q_{ψ_L}	Q_{ψ_R}	Q_η	Q_χ	Q_σ	necessary interactions
C1	-1/15	7/15	8/15	2/15	-2/5	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma, \Phi^\dagger\eta\chi^*\sigma, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C2	-1/9	1/9	8/9	2/9	-2/3	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L, \Phi^\dagger\eta\chi^*\sigma, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C3	-1/9	7/9	2/9	2/9	-2/3	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma, \Phi^\dagger\eta\chi^*, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C4	-1/3	-5/3	8/3	2/3	-2	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma^*, \Phi^\dagger\eta\chi^*\sigma, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C5	-1/3	7/3	-4/3	2/3	-2	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma, \Phi^\dagger\eta\chi^*\sigma^*, \Phi^\dagger\eta\chi^2, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C6	1/9	5/9	4/9	-2/9	-2/3	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma, \Phi^\dagger\eta\chi^*\sigma, \Phi^\dagger\eta\chi^2, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$
C7	-1/3	1/3	2/3	2/3	-2	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L, \Phi^\dagger\eta\chi^*, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma$
C8	1/3	5/3	-2/3	-2/3	-2	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L\sigma, \Phi^\dagger\eta\chi^*, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$
C9	1/3	-1/3	4/3	-2/3	-2	$\bar{E}\psi_R\tilde{\eta}, \bar{\psi}_R\psi_L, \Phi^\dagger\eta\chi^*\sigma, \Phi^\dagger\eta\chi^2, \bar{\psi}_L^c\psi_L\chi, \chi^3\sigma^*$

TABLE VI: Possible charge assignments of the extended two-loop model II. The last column contains the necessary interactions to generate neutrino masses at two-loop level. Only assignments C5, C6 and C9 reproduce the original two-loop model II because of the $\Phi^\dagger\eta\chi^2$ term.

the other assignments do not replicate the original model, the neutrino masses are produced at two-loop level and the DM candidate is included.

D. Three-loop \mathbb{Z}_4 Model

Based on the method we have developed, one can readily build a radiative neutrino mass model with an unbroken \mathbb{Z}_{N+1} symmetry at N-loop level, and the scalar interaction $\chi^{N+1}\sigma$ (or $\chi^{N+1}\sigma^*$) is essential for constructing the loop diagrams of the neutrino masses in the corresponding $U(1)_{B-L}$ extended model. As an example, here we demonstrate a $U(1)_{B-L}$ extended three-loop model in which a dimensional five operator $\chi^4\sigma$ (or $\chi^4\sigma^*$) is introduced.⁵ However, in order to have a UV

⁵ For $N > 3$, the observed neutrino masses may not be satisfied due to the loop-suppression factor $(4\pi)^{2N}$. That is to say, the derived neutrino masses could be too small to reproduce the observations. The first phenomenologically valid four-loop neutrino mass model has recently been proposed in Ref. [46, 47].

	E	Φ	N_R^c	Σ_L	Σ_R^c	ψ_L	ψ_R^c	χ	s	σ
$SU(2)_L$	2	2	1	2	2	1	1	1	1	1
$U(1)_Y$	$-1/2$	$1/2$	0	$-1/2$	$1/2$	0	0	0	0	0
$U(1)_{B-L}$	-1	0	Q_N	Q_{Σ_L}	Q_{Σ_R}	Q_{ψ_L}	Q_{ψ_R}	Q_χ	Q_s	Q_σ
Spin J	$1/2$	0	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	0	0	0

TABLE VII: Charge assignments of the fermions and scalars in the $U(1)_{B-L}$ extended three-loop model.

complete theory, we add one more complex scalar s to induce $\chi^4\sigma$ through the trilinear scalar interactions χ^2s and $s^2\sigma$ when s is integrated out. To fulfill the model, we employ the particle contents and the charge assignments as displayed in Table VII. The interactions for generating the neutrino masses at three-loop level are then as follows

$$\mathcal{L} \supset \overline{E}\Sigma_R\chi^* \left| \begin{array}{c} \overline{N_R^c}N_R \\ \overline{N_R^c}N_R\sigma \\ \overline{N_R^c}N_R\sigma^* \end{array} \right| \left| \begin{array}{c} \overline{\Sigma_R}\Sigma_L \\ \overline{\Sigma_R}\Sigma_L\sigma \\ \overline{\Sigma_R}\Sigma_L\sigma^* \end{array} \right| \left| \begin{array}{c} \overline{\psi_R}\psi_L \\ \overline{\psi_R}\psi_L\sigma \\ \overline{\psi_R}\psi_L\sigma^* \end{array} \right| \left| \begin{array}{c} \overline{\Sigma_L}\psi_R\tilde{\Phi} \\ \overline{\Sigma_R}\psi_L\tilde{\Phi} \end{array} \right| \left| \begin{array}{cc} \overline{N_R}\psi_L\chi & \overline{N_L}\psi_R\chi \\ \overline{N_R}\psi_L\chi^* & \overline{N_L}\psi_R\chi^* \end{array} \right| \left| \begin{array}{cc} \chi^2s & \chi^2s^* \\ \chi^2s\sigma & \chi^2s^*\sigma \\ \chi^2s\sigma^* & \chi^2s^*\sigma^* \end{array} \right| \left| \begin{array}{c} s^2\sigma \\ s^2\sigma^* \end{array} \right| + \text{h.c.} . \quad (7)$$

At least one interaction from each column is required to obtain the neutrino masses. One possible topological diagram of the neutrino mass generation is shown in Fig. 4.⁶ By integrating out the complex scalar field s , the topology of the neutrino mass diagram is exactly the same as the one in the KNT model [18]. We give one possible B–L charge assignment with the necessary interactions in Table VIII. The other assignments can be easily found by the same approach as mentioned above. After the $U(1)_{B-L}$ symmetry breaking, the remaining symmetry $\mathbb{Z}_4 \equiv (-1)^{N_{B-L}}$ ensures

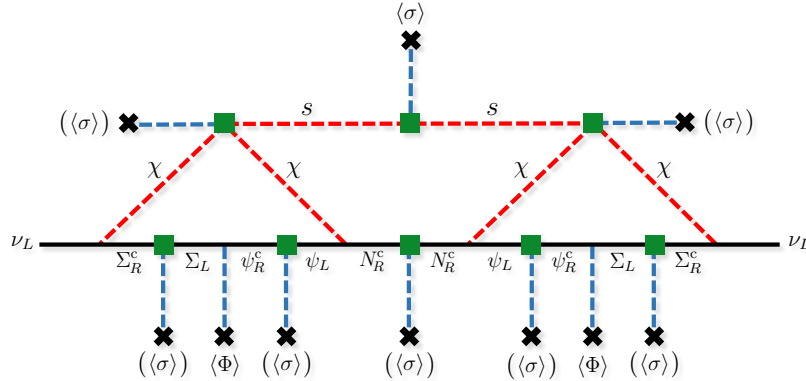


FIG. 4: Topology of the neutrino mass generation in the three-loop model.

⁶ The other two possible topological diagrams can be obtained by different contractions of χ in Fig. 4 and should be added if one derives the complete neutrino mass formula for phenomenological discussions. Furthermore, depending on how the B–L charge is assigned, one may have accidental Yukawa interaction $\overline{N_L}\psi_R\chi^*$ (analogy to the assignments B5-B7) or trilinear scalar coupling χ^2s (similar to the assignments C5, C6 and C9) to generate the neutrino mass diagrams (e.g. see the right-panel diagrams of Fig. 2 and Fig. 3).

Q_N	Q_{Σ_L}	Q_{Σ_R}	Q_{ψ_L}	Q_{ψ_R}	Q_χ	Q_s	Q_σ
1/8	-9/16	13/16	-5/16	9/16	3/16	-1/8	-1/4

TABLE VIII: One possible B–L charge assignment of the new particles in the $U(1)_{B-L}$ extended three-loop model. The necessary interactions for generating the neutrino masses at three-loop level are $\overline{E}\Sigma_R\chi^*$, $\overline{N}_R^c N_R \sigma^*$, $\overline{\Sigma}_R \Sigma_L \sigma$, $\overline{\Sigma}_L \psi_R \tilde{\Phi}$, $\overline{N}_R \psi_L \chi$, $\chi^2 s \sigma$ and $s^2 \sigma^*$.

the lightest \mathbb{Z}_4 odd particle in this model to be a DM candidate while N_R^c was the unique DM candidate in the KNT model.

III. ANOMALY CANCELLATION

All of the models we have discussed so far include gauge anomalies which should not appear to be consistent with the gauge symmetry at quantum level. This is problematic, however, these anomalies can be cancelled by introducing some new exotic fermions [34–36]. For the most models described above, strictly for the one-loop \mathbb{Z}_2 model, two-loop \mathbb{Z}_3 model II and B2, B7, B9 assignments of two-loop \mathbb{Z}_3 model I, it is sufficient to impose only two kinds of the anomaly cancellation conditions for $[\text{gravity}]^2 \otimes U(1)_{B-L}$ and $[U(1)_{B-L}]^3$ to cancel all the anomalies since all the new fermions in these models are vector-like or neutral under the SM gauge group. However, for the other cases, since some pairs of the $SU(2)_L$ doublet fermions are introduced and they are non-vector-like in terms of $U(1)_{B-L}$ symmetry, additional anomaly cancellation conditions for $[SU(2)_L]^2 \otimes U(1)_{B-L}$, $[U(1)_{B-L}]^2 \otimes U(1)_Y$ and $U(1)_{B-L} \otimes [U(1)_Y]^2$ must be imposed. Nevertheless, all the anomalies can basically be cancelled by adding some new exotic fermions. Here we give an example of the anomaly cancellations for the one-loop \mathbb{Z}_2 model and two-loop \mathbb{Z}_3 model II.

For anomaly cancellations, we introduce n_ξ pairs of (ξ_L, ξ_R^c) , n_ζ pairs of (ζ_L, ζ_R^c) and n_λ generations of λ_L as shown in Table IX where all the fermions transform as singlets under the SM gauge group. In addition, we assume that mass terms of the exotic fermions are induced by the VEV $\langle \sigma \rangle$ in order to reduce the free parameters displayed in Table IX. To be specific, $U(1)_{B-L}$ charges of the exotic fermions are assigned such that the terms $\sigma^* \overline{\xi_R} \xi_L$, $\sigma \overline{\zeta_R} \zeta_L$ and $\sigma^* \overline{\lambda_L} \lambda_L$ are allowed. This implies that the following relations are imposed: $Q_{\xi_L} + Q_{\xi_R} = Q_\sigma$, $Q_{\zeta_L} + Q_{\zeta_R} = -Q_\sigma$ and $2Q_{\lambda_L} = Q_\sigma$. One more point one should note is that the charges of the exotic fermions have to be assigned not to affect to the aforementioned models. For instance, we request the charge $Q_{B-L} \neq 1$, otherwise one can have Dirac Yukawa coupling, and the neutrino masses are generated by the canonical seesaw mechanism at tree-level.

	ξ_L	ξ_R^c	ζ_L	ζ_R^c	λ_L
$U(1)_{B-L}$	Q_{ξ_L}	Q_{ξ_R}	Q_{ζ_L}	Q_{ζ_R}	Q_{λ_L}
Generations	n_ξ		n_ζ		n_λ

TABLE IX: Exotic fermions added for anomaly cancellations and their B–L charge and generations.

The conditions of anomaly cancellations for $[\text{gravity}]^2 \otimes \text{U}(1)_{\text{B-L}}$ and $[\text{U}(1)_{\text{B-L}}]^3$ are given by

$$\sum Q_{\text{B-L}} = \sum Q_{\text{B-L}}^3 = 0, \quad (8)$$

where the summation is taken over all the fermions included in the models. For the one-loop \mathbb{Z}_2 model and two-loop \mathbb{Z}_3 model II, these anomaly cancellation conditions are explicitly given by

$$-3 + \sum Q_{\text{model}} + (n_\xi - n_\zeta + \frac{1}{2}n_\lambda)Q_\sigma = 0, \quad (9)$$

$$-3 + \sum Q_{\text{model}}^3 + [(n_\xi - n_\zeta) + \frac{1}{8}n_\lambda]Q_\sigma^3 - 3(n_\xi Q_{\xi_L} Q_{\xi_R} - n_\zeta Q_{\zeta_L} Q_{\zeta_R})Q_\sigma = 0, \quad (10)$$

where $\sum Q_{\text{model}}$ and $\sum Q_{\text{model}}^3$ are summed over the new fermions for neutrino masses in each model and not the exotic fermions for anomaly cancellations. These are explicitly given by

$$\sum Q_{\text{model}} = \begin{cases} n_N Q_N & \text{for one-loop } \mathbb{Z}_2 \text{ model} \\ n_\psi (Q_{\psi_L} + Q_{\psi_R}) & \text{for two-loop } \mathbb{Z}_3 \text{ model II} \end{cases}, \quad (11)$$

and

$$\sum Q_{\text{model}}^3 = \begin{cases} n_N Q_N^3 & \text{for one-loop } \mathbb{Z}_2 \text{ model} \\ n_\psi (Q_{\psi_L}^3 + Q_{\psi_R}^3) & \text{for two-loop } \mathbb{Z}_3 \text{ model II} \end{cases}, \quad (12)$$

with the number of generations of N and ψ represented by n_N and n_ψ , respectively. The $\text{U}(1)_{\text{B-L}}$ charges Q_N , Q_{ψ_L} and Q_{ψ_R} are listed in Table II and VI for each model. One can find some solutions satisfying these conditions. An example of B–L charge assignments and number of generations being consistent with the gauge anomalies for each model is given in Table X and XI in Appendix.

Even if we add these new fermions for anomaly cancellations, the discussion of the \mathbb{Z}_2 or \mathbb{Z}_3 symmetry which remains after the $\text{U}(1)_{\text{B-L}}$ symmetry breaking does not change because the exotic fermions introduced for anomaly cancellations are completely separated from these models, while they may affect to the discussion of Z' width below. One more comment should be stated on the new exotic fermions. After the spontaneous $\text{U}(1)_{\text{B-L}}$ symmetry breaking, a different kind of discrete symmetry would be left in this new sector. This entails that a new DM candidate emerges and thus the models have two-component DM which can interact with each other through Z' and the Higgs bosons. When the relic density of DM is calculated, one should note that the number density of two-component DM can be changed by conversion processes. Thereby, one has to generally solve the coupled Boltzmann equation for two-component DM in order to compute the DM relic abundance [48–50]. Moreover, the new exotic fermions may associate with the other DM phenomenology such as detection properties. For example, recoil-energy spectrum of number of event for elastic scattering with nuclei can be discriminable if the masses of the two DM components are non-degenerate [51, 52]. Furthermore, double monochromatic gamma-ray or neutrino lines at two distinct energies can be predicted as indirect detection signal due to the mass splitting between the two DM components while such a double peak may not be a clear signal of multi-component DM since it is possible to generate similar gamma-ray or neutrino lines even for single-component DM through different annihilation channels [53]. However, exploration of phenomenology of multi-component DM is beyond the scope of the paper.

IV. Z' WIDTH AS A PROBE OF THE GAUGED DISCRETE SYMMETRY

A search for a new gauge boson is the most powerful way to probe new physics beyond the SM. The models we studied the above predicts new neutral gauge boson Z' , which may be observed at the LHC (and/or FCC). Unknown U(1) charges for the SM particles can be tested by analyzing the different decay modes of the new boson. The universality among fermion generation would be easily checked by clean di-lepton (di-electron and di-muon) channels, and the quark-lepton universality can also be examined by quark flavor tagging. Therefore, the $U(1)_{B-L}$ gauge symmetry can be reconstructed by these measurements.

Furthermore, the total width of Z' is observable by looking at the invariant mass distribution, which potentially gives us an additional information about the charges of new particles. We here define a ratio of Z' width,

$$\mathcal{R}_{\text{new}} = \frac{\Gamma(Z' \rightarrow \text{all}) - \Gamma(Z' \rightarrow \text{SM})}{\Gamma(Z' \rightarrow \mu\mu)/2} \approx \frac{\sum_{\text{new}} Q_{B-L}^2}{(Q_{\mu_L}^2 + Q_{\mu_R}^2)/2} = \sum_{\text{new}} Q_{B-L}^2. \quad (13)$$

where Q_{μ_L} and Q_{μ_R} are the B-L charges of left-handed and right-handed muon, respectively. The summation runs over all the new particles with a mass smaller than $M_{Z'}/2$. This quantity \mathcal{R}_{new} is a rational number (if Z' is sufficiently heavier than the daughter new particles), which characterizes the property of the original gauged $U(1)_{B-L}$ gauge symmetry. Measuring \mathcal{R}_{new} , we can, in principle, distinguish the resultant discrete symmetries for DM stability and the radiative neutrino models in each category.

Although we have considered the $U(1)_{B-L}$ gauge extension of the models with the radiative neutrino masses, the $U(1)_L$ gauge group can also be used for the extension since we only focused on the lepton sector for loop-induced neutrino masses. In fact, the $U(1)_B \otimes U(1)_L$ symmetry has been suggested as an extension of the SM [54–56].⁷ This symmetry is motivated since the dimensional six operator inducing proton decay $QQQE/\Lambda^2$ can be forbidden, which is allowed for $U(1)_{B-L}$ symmetry where Q is the $SU(2)_L$ doublet quark and Λ is the cut-off scale. The cut-off scale Λ should be larger than GUT scale to be consistent with the lifetime of proton $\tau_p \gtrsim 10^{32}$ yrs. This implies that any particle in intermediate scale between electroweak (TeV) scale and GUT scale does not exist in the case of the $U(1)_{B-L}$ symmetry. On the other hand, the cut-off scale Λ can be lower than GUT scale for the case of the $U(1)_B \otimes U(1)_L$ symmetry since operators inducing proton decay become higher than the mass dimension six. This means that new particles at intermediate scale can be allowed. The $U(1)_B \otimes U(1)_L$ symmetry would also be applied to our model building instead of $U(1)_{B-L}$ symmetry. Consequently, only the Z' phenomenology would be different from the models with $U(1)_{B-L}$.

V. SUMMARY AND CONCLUSIONS

We have presented a prescription for systematically classifying the gauged $U(1)_{B-L}$ extended models for the radiative neutrino mass generation and the DM stability. The tiny neutrino masses

⁷ $U(1)_{B+L}$ symmetry has also been explored in very recently paper [57].

are naturally explained by the loop-suppression of the radiative seesaw mechanism, while the DM stability is automatically maintained by the residual symmetry of the spontaneous $U(1)_{B-L}$ symmetry breaking. These models are classified by the identifications (the insertion of the VEV) of the $B-L$ breaking vertices in the prototype models for the loop-induced neutrino masses with a discrete symmetry. We found five independent models for the one-loop \mathbb{Z}_2 model, while nine independent models for each two-loop \mathbb{Z}_3 model. This procedure is easily extended to the models based on the higher loop diagrams and \mathbb{Z}_N symmetry. A certain ratio of the width of the new gauge boson related to the $U(1)_{B-L}$ symmetry is a good discriminant of these models, which can be tested at future high energy colliders.

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Appendix: Fermion Contents and Charges for Anomaly Cancellations

An example of $B-L$ charge assignments for anomaly cancellations is given for the one-loop \mathbb{Z}_2 model in Table X and for the two-loop \mathbb{Z}_3 model II in Table XI.

	Q_N	Q_{ξ_L}	Q_{ξ_R}	Q_{ζ_L}	Q_{ζ_R}	Q_{λ_L}	n_N	n_ξ	n_ζ	n_λ
A1	1/2	1/10	-11/10	6/5	-1/5	—	4	2	3	—
A2	1/4	-1/8	-3/8	15/16	-7/16	—	6	1	4	—
A3	-1/2	1/6	5/6	-3/4	-1/4	—	4	9	4	—
A4	0	7/3	-13/3	8/3	-2/3	-1	3	1	4	3
A5	0	-1/9	-5/9	-1/9	7/9	-1/3	3	1	7	3

TABLE X: Assignments and number of generations for the one-loop \mathbb{Z}_2 model.

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	Q_{ψ_L}	Q_{ψ_R}	Q_{ξ_L}	Q_{ξ_R}	Q_{ζ_L}	Q_{ζ_R}	Q_{λ_L}	n_ψ	n_ξ	n_ζ	n_λ
C1	$-1/15$	$7/15$	0	$-2/5$	$11/15$	$-1/3$	$-1/5$	4	4	8	1
C2	$-1/9$	$1/9$	$-1/6$	$-1/2$	$3/4$	$-1/12$	$-1/3$	3	2	8	3
C3	$-1/9$	$7/9$	$2/3$	$-4/3$	$11/9$	$-5/9$	$-1/3$	3	4	6	1
C4	$-1/3$	$-5/3$	$1/6$	$-13/6$	$7/4$	$1/4$	-1	4	2	8	1
C5	$-1/3$	$7/3$	$4/3$	$-10/3$	$4/3$	$2/3$	-1	3	1	1	3
C6	$1/9$	$5/9$	$11/9$	$-17/9$	$5/3$	-1	$-1/3$	4	1	2	1
C7	$-1/3$	$1/3$	$11/3$	$-17/3$	$10/3$	$-4/3$	-1	3	1	4	3
C8	$1/3$	$5/3$	$7/3$	$-13/3$	5	-3	-1	3	3	2	1
C9	$1/3$	$-1/3$	5	-7	4	-2	-1	3	1	4	3

TABLE XI: Assignments and number of generations for the two-loop \mathbb{Z}_3 model II.

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